Understanding and predicting the Brewer-Dobson Circulation

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Special thanks to the U.S. National Science Foundation
Recent Ozone

OMI Total Ozone Jan 10, 2014

Dobson Units

Dark Gray < 100 and > 500 DU
Recent Ozone: the Brewer-Dobson Circulation

Dobson, Harrison, and Lawrence [1929]
Recent Ozone: the Brewer-Dobson Circulation

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Dobson, Harrison, and Lawrence [1929]
Recent Ozone: the Brewer-Dobson Circulation

The only way in which we could reconcile the observed high ozone concentration in the Arctic in spring and the low concentration within the Tropics, with the hypothesis that the ozone is formed by the action of sunlight, would be to suppose a general slow poleward drift in the highest atmosphere with a slow descent of air near the Pole. Such a current would carry ozone formed in low latitudes to the Pole and concentrate it there. If this were the case the

Dobson, Harrison, and Lawrence [1929]
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By A. W. BREWER, M.Sc., A.Inst.P.

(Manuscript received 23 February 1949)

Isotherms over the Globe

Fig. 5. A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause.
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\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} - f v = - \frac{1}{\rho} \frac{\partial p}{\partial x}
\]
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\[ \frac{\partial \bar{u}}{\partial t} - f \bar{v} = - \frac{\partial}{\partial y} \bar{u}'v' \]
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\frac{\partial \bar{u}}{\partial t} - f \bar{v} &=& - \frac{\partial}{\partial y} \bar{u} \bar{v}'
\end{eqnarray*}
\]

"polar vortex catastrophe"
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\]

\[
\frac{\partial \bar{u}}{\partial t} - f\bar{v} = -\frac{\partial}{\partial y} u'v'
\]

\[
\frac{\partial \bar{u}}{\partial t} - f \left( \bar{v} - \frac{\partial}{\partial z} \frac{u'\theta'}{\theta_p} \right) = \frac{\partial}{\partial y} \left( -u'v' \right) + \frac{\partial}{\partial z} \frac{f v'\theta'}{\theta_p}
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\[
\begin{align*}
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\frac{\partial \bar{u}}{\partial t} - f \bar{v}^* &= \nabla \cdot \mathbf{F}
\end{align*}
\]

Eliassen and Palm, 1961

Andrews and McIntyre, 1976
Questions

• What drives the Brewer-Dobson Circulation?

• How will the Brewer-Dobson Circulation respond to anthropogenic forcing?
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• How will the Brewer-Dobson Circulation respond to anthropogenic forcing?

Focus will be on the residual circulation, which transports mass across isentropic surfaces.

Tracer transport — which Brewer and Dobson actually observed — also depends critically on mixing along isentropes … please see Alan Plumb and/or me over coffee / beer!
Questions

• What drives the Brewer-Dobson Circulation?
  (Which waves are responsible for balancing the Coriolis torque?)
Questions

- What drives the Brewer-Dobson Circulation?

(Which waves are responsible for balancing the Coriolis torque?)

(a) Annual mean upward mass flux at 70 hPa

[CCMVal2 Report, Butchart et al. 2011]
Questions

• How will the Brewer-Dobson Circulation respond to anthropogenic forcing?
  • Models uniformly predict that it will increase [e.g. Butchart et al. 2010],
    but can’t be validated w/ available measurements [e.g. Garcia et al. 2011].
  • Do we understand why?
Questions

• How will the Brewer-Dobson Circulation respond to anthropogenic forcing?
  • Models uniformly predict that it will increase \([\text{e.g. Butchart et al. 2012}]\), but can’t be validated with available measurements \([\text{e.g. Garcia et al. 2011}]\).
  • Do we understand why? Yes \([\text{e.g. Shepherd and McLandress 2011}]\), but…

(c) Annual mean mass flux trend at 70 hPa, 2000-2049

<table>
<thead>
<tr>
<th>Model</th>
<th>Mass Flux (10^9 kg s^-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCMVal2 Report</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
</tr>
<tr>
<td>Rossby waves</td>
<td></td>
</tr>
<tr>
<td>orographic GW</td>
<td></td>
</tr>
<tr>
<td>non-orographic GW</td>
<td></td>
</tr>
</tbody>
</table>

[CCMVal2 Report]
Questions

• What drives the Brewer-Dobson Circulation?

• How will the Brewer-Dobson Circulation respond to anthropogenic forcing?

Interactions between Rossby and gravity wave driving complicate the answer to these questions.
What drives the Brewer-Dobson Circulation?
Downward Control [Haynes et al. 1991]

\[ \frac{\partial \bar{u}}{\partial t} - \bar{v}^* \left( f - \frac{\partial \bar{u}}{\partial y} \right) + \bar{w}^* \frac{\partial \bar{u}}{\partial z} = \mathcal{F} \text{ zonal mean torque} \]

(Transmformed Eulerian Mean momentum equation)
Downward Control [Haynes et al. 1991]

\[
\frac{\partial \overline{u}}{\partial t} - \overline{v}^* \left(f - \frac{\partial \overline{u}}{\partial y}\right) + \overline{w}^* \frac{\partial \overline{u}}{\partial z} = \mathcal{F} \quad \text{zonal mean torque}
\]

steady state
Downward Control [Haynes et al. 1991]

\[
\frac{\partial \bar{u}}{\partial t} - \bar{v}^* \left( f - \frac{\partial \bar{u}}{\partial y} \right) + \bar{w}^* \frac{\partial \bar{u}}{\partial z} = \mathcal{F} \quad \text{zonal mean torque}
\]

QG (neglect relative vorticity)
Downward Control [Haynes et al. 1991]

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} - \bar{v}^* \left( f - \frac{\partial \bar{u}}{\partial y} \right) + \bar{w}^* \frac{\partial \bar{u}}{\partial z} &= F \\
\text{zonal mean torque}
\end{align*}
\]

Coriolis force must balance torque

\[
\bar{v}^* = -\frac{F}{f}
\]
Downward Control [Haynes et al. 1991]

\[ \frac{\partial \bar{u}}{\partial t} - \bar{v}^* \left( f - \frac{\partial \bar{u}}{\partial y} \right) + \bar{w}^* \frac{\partial \bar{u}}{\partial z} = F \]

zonal mean torque

Coriolis force must balance torque

\[ \bar{v}^* = -\frac{F}{f} \]
Which waves contribute to the zonal mean torque?

\[ \mathcal{F} = \nabla \cdot \mathbf{F} + G_{OGW} + G_{NOGW} \]
Which waves contribute to the zonal mean torque?

\[ \mathcal{F} = \nabla \cdot F + G_{OGW} + G_{NOGW} \]

Eliassen-Palm flux divergence: Rossby waves, planetary and synoptic; fairly well observed, resolved in models.
Which waves contribute to the zonal mean torque?

\[ F = \nabla \cdot F + G_{OGW} + G_{NOGW} \]

Orographic gravity waves, of scale 10-1000 km, generated in stratified flow over topography; marginally observed, parameterized in models.
Which waves contribute to the zonal mean torque?

\[ \mathcal{F} = \nabla \cdot \mathbf{F} + G_{OGW} + G_{NOGW} \]

- non-orographic gravity waves: generated via convection, frontal instabilities (thus have non-zero phase speed), less well observed, parameterized in models.
Which waves contribute to the zonal mean torque?

\[ \mathcal{F} = \nabla \cdot F + G_{OGW} + G_{NOGW} \]

using downward control, one can partition the circulation

\[ \psi = \psi_{EPFD} + \psi_{OGW} + \psi_{NOGW} \]
Which waves contribute to the zonal mean torque?

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using downward control, partition the circulation

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implicit assumption: the wave forcings are independent
The JJA Residual Circulation in ECHAM6

residual mean mass streamfunction (kg s\(^{-1}\))

latitude

pressure (hPa)
The JJA Residual Circulation in ECHAM6
Breaking down the streamfunction
Breaking down the streamfunction

residual mean streamfunction (kg s$^{-1}$)

EPFD streamfunction (kg s$^{-1}$)

EPFD (m s$^{-1}$ day$^{-1}$)
Breaking down the streamfunction

- EPFD, OGWD (m s\(^{-1}\) day\(^{-1}\))
- Residual mean streamfunction (kg s\(^{-1}\))
- OGWD
Breaking down the streamfunction

residual mean streamfunction (kg s\(^{-1}\))

EPFD, OGWD (m s\(^{-1}\) day\(^{-1}\))

EPFD, OGWD, NOGWD \(\psi\) (kg s\(^{-1}\))
Puzzle pieces fit together to provide a smooth circulation!
This decomposition of the BDC is used to assess the roles of each type of wave driving.
This decomposition of the BDC is used to assess the roles of each type of wave driving.
What drives the Brewer-Dobson Circulation?

(a) Annual mean upward mass flux at 70 hPa

[CCMVal2 Report, Chpt 4]
Why do the models agree more on the total circulation than on the components?

How do the components fit together so nicely to produce a smooth circulation?
An idealized Atmospheric GCM

- dry primitive equations on the sphere
- Newtonian relaxation of temperature to radiative-convective equilibrium profile [Held and Suarez 1994; Polvani and Kushner 2002]
- Simple large scale topography [Gerber and Polvani, 2009]
- Alexander and Dunkerton [1999] non-orographic gravity wave drag
- Pierrehumbert [1987] orographic gravity wave drag

[Cohen et al. 2013]
Two experiments:
Perturb the Orographic Gravity Wave Drag

Model A: positive correlation

Model B: negative correlation

[Cohen et al. 2013]
Impact of differences in OGW configuration

OGW driving (10⁹ N) “positive correlation”

OGW driving (10⁹ N) “negative correlation”

[Cohen et al. 2013]
Impact on BDC

Cohen et al. 2013
Impact on BDC

difference in OGW driving ($10^9$ N)

[CoHer et al. 2013]
Impact on BDC

difference in OGW driving ($10^9$ N)

difference $\psi^*$, downward control ($10^9$ kg/s)

actual difference $\psi^*$ ($10^9$ kg/s)
Compensation by the resolved waves

difference EP flux divergence (10^9 N)

difference $\psi^*$, downward control (10^9 kg/s)

actual difference $\psi^*$ (10^9 kg/s)

OGWD
What “drives” the BDC?

Residual Mean Streamfunction at 70 hPa

Model A

Model B

[80 60 40 20 0 20 40 60 80] latitude

mass streamfunction (10^9 kg s^{-1})

Direct
EPFD+GWD
EPFD
NOGWD
OGWD

[80 60 40 20 0 20 40 60 80] latitude

mass streamfunction (10^9 kg s^{-1})

Direct
EPFD+GWD
EPFD
NOGWD
OGWD

[80 60 40 20 0 20 40 60 80] latitude

mass streamfunction (10^9 kg s^{-1})

Direct
EPFD+GWD
EPFD
NOGWD
OGWD

[Cohen et al. 2013]
Implication of compensation for BDC driving...

(a) Annual mean upward mass flux at 70 hPa

- BDC
- EPFD+GWD
- EPFD
- NOGWD
- OGWD

Mass Flux (10⁹ kg s⁻¹)

Model A
Model B
What is going on here?
What is going on here?

When I find myself in times of trouble, Father Hoskins comes to me, speaking words of wisdom …

PV … PV!
What is going on here?

When I find myself in times of trouble, Father Hoskins comes to me, speaking words of wisdom …

PV … PV!

(That is, how do the wave forcings affect the potential vorticity.)
Back to Basics: Haynes et al. 1991
(Near) steady response to a localized torque

zonal wind

streamfunction $\psi$

(i)

(j)
For what torques is the circulation reasonable?

zonal wind
For what torques is the circulation reasonable?

QG Potential Vorticity

\[
\bar{q}_y = \beta - \bar{u}_{yy} + f \frac{\bar{\theta}_y}{\bar{\theta}_p}
\]
For what torques is the circulation reasonable? Stability depends critically on meridional scale.

Zonal wind amplitude $A$, meridional scale $L$.

**QG Potential Vorticity**

$$\overline{q_y} = \beta - \overline{u_{yy}} + f \frac{\overline{\theta y}}{\overline{\theta p}}$$

$$\overline{u} \sim \frac{A}{L^2}$$
For what torques is the circulation reasonable?

Stability depends critically on meridional scale $L$.

**QG Potential Vorticity**

$$\bar{q}_y = \beta - \bar{u}_{yy} + f \frac{\bar{\theta}_y}{\bar{\theta}_p}$$

$$\bar{u} \sim \frac{A}{L^2}$$

For $L << L_R$

amplitude $A$, meridional scale $L$

perturbation to PV gradient $\sim \frac{A}{L^4}$
Stability of the circulation for a compact torque
Stability of the circulation for a compact torque

![Graph showing stability regions for different wave driving amplitudes and meridional length scales.](image)
Stability of the steady state for compact torque

![Graph showing stability limits for compact torque]
Test the prediction

amplitude of wave driving $A$ (ms$^{-2}$)

meridional length scale $L$ (degrees)

unstable

stable

Note that the factor $L_d$ stands for the contribution of a perturbed QG-PV meridional gradient to instability. For a gradient in the relative vorticity $Q_y$, the necessary condition for instability is that

$$
Q_y = \beta
$$

Here, the critical amplitude for instability decreases rapidly with the meridional scale of the perturbation, which is extremely sensitive to the vertical scale of the wave forcing. The larger the vertical scale of the wave forcing height, the smaller the critical amplitude is inversely proportional to the square of the vertical scale.

In the limit the amplitude must be fairly large to estimate the critical amplitude $L_d^c$.

In an OGW parameterization, the instability is quite likely to be satisfied.

Following the downward control derivation, using the existing meridional gradient, Equation (6) can be separated for a basic-state balance governed by

$$
\begin{align*}
\frac{d}{dt} Q &= \nabla \cdot \mathbf{v}^* - \nabla \cdot \mathbf{w}^* \\
0 &= \nabla \cdot \mathbf{w}^* + \nabla \cdot \mathbf{v}^*
\end{align*}
$$

and where the torque is centered around $2u_t$.

Note that the factor $L_d$ is large compared to the Rossby radius of deformation $L_d^c$.

In words, the necessary condition for instability is that

$$
Q_y \gg 1
$$

Under the assumption that the wave forcing is linearly amplitude of wave driving.

We next test the hypothesis that compensation is related to instability in our AGCM.
Test the prediction

![PV Gradient](image)

- **narrow**
  - L=2.5
  - L=5.0
  - L=7.5
  - L=10.0
  - L=12.5
  - L=15.0
  - L=17.5
  - L=20.0
  - L=22.5
  - L=25.0

- **wide**
Breaking down the streamfunction

EPFD, OGWD (m s$^{-1}$ day$^{-1}$)

residual mean streamfunction (kg s$^{-1}$)

OGWD
Interaction between wave driving suggest that the “forcings” are somewhat fungible.

\[ \mathcal{F} = \nabla \cdot F + G_{OGW} + G_{NOGW} \]

\[ \psi = \psi_{EPFD} + \psi_{OGW} + \psi_{NOGW} \]
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\[ \mathcal{F} = \nabla \cdot F + G_{OGW} + G_{NOGW} \]

\[ \psi = \psi_{EPFD} + \psi_{OGW} + \psi_{NOGW} \]
Compensation makes total circulation more robust than components [Sigmond and Shepherd, 2014]

Fig. 4. The mass stream function $\Psi$ at 70 hPa for NH winter (DJF) as a function of $G$ (first column) at the turnaround latitude and (second column) the difference between $\Psi$ at the turnaround latitude and 52$^\circ$N for (first row) $1\times\text{CO}_2$ and (second row) the response to climate change.

The diagram shows the mass stream function $\Psi$ as a function of the intensity of parameterized wave driving (parameter $G$). The graph includes four lines representing different components:
- **Direct**: Black line
- **OGW**: Red line
- **OGW+EP**: Gray line
- **EP**: Blue line

The x-axis represents the intensity of parameterized wave driving, ranging from 0 to 1, and the y-axis represents the mass stream function $\Psi$ in units of $(\text{kg m}^{-1} \text{s}^{-1})$.
Uncertainty in “forcing” increases with future trends
... but compensation may affect response to CO₂

[Shepherd and McLandress 2011]
Impact of GW depends on basic state of the model [Sigmond and Shepherd, 2014]

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The intensity of parameterized wave driving (parameter $G$)
Impact of GW depends on basic state of the model [Sigmond and Shepherd, 2014]
Tuning of the basic state influences the relative role of wave forcings in climate response
A potential vorticity, surf zone perspective

Action of Rossby waves is to mix potential vorticity in the surf zone between the polar vortex and tropical stratosphere.

[McIntyre and Palmer, 1983]
A potential vorticity, surf zone perspective

Gravity wave driving inside surf zone will have limited impact on the BDC.

More likely for stationary OGW, which break at same critical levels as stationary Rossby waves

[Cohen et al. 2014]
A potential vorticity, surf zone perspective

Gravity wave driving outside surf zone likely to have large impact on the BDC.

More likely for NOGW, which can modify polar vortex.

[Cohen et al. 2014]
Anthropogenic forcing modifies surf zone [Shepherd and McLandress 2011]

Expansion of subtropical jets raises critical level for wave breaking.

Stratosphere is shrinking, lifting the surf zone!
Conclusions

• The Brewer-Dobson Circulation is wave driven, but defining the precise role of Rossby vs. gravity waves is problematic, given their interactions.
  • resolved waves clearly dominant in the stratosphere: mixing PV
  • impact of gravity waves, particularly non-orographic waves, may largely be indirect, by shaping the Rossby wave forcing
  • intermodel differences in wave driving likely reflect tuning, not fundamental limitations in our understanding

• Models accurately simulate the current BDC (albeit with tuning), and robustly predict an increase in the future
  • differences in role of GW vs. resolved waves may be a red herring
  • Mechanism of rising critical latitudes (i.e. a shrinking of the stratosphere) is robust

• Idealized GCMs provide a bridge to connect theoretical insights with the observed and modeled Brewer-Dobson Circulation
Fig. 3. The conceptual model for the stratospheric surf zone. Planetary-wave breaking mixes the PV (dashed blue) and drag the flow away from the background PV (in solid red). The total wave breaking is constrained to be equal to the absolute value of the area between the two curves.
Experiment to separate mixing and instability pathways towards compensation
Fig. 10. The ensemble-mean transient response of the flow to the torque placed in the SH stratosphere (a) and (c) and in the NH stratospheric surf zone (b) and (d), evaluated at the region surrounding the torque. In (a) and (b) the ensemble-mean meridional derivative of the PV (red), the ensemble-mean EPFD (blue) and the ensemble-mean change in the maximum zonal wind (green), all scaled by their climatological values for comparison, as a function of time (the zonal wind change is additionally scaled by 10 to ease on comparison). In (c) and (d) the ensemble-mean (blue) and long-time mean (dashed black) compensation as a function of time. The standard deviation was verified to be negligible, using the Bootstrap method.
Amplifying effect of NOGW

Fig. 12. Analysis of a non-compensating integration, where the perturbation is defined to be the difference between the NOGWD and the Rayleigh drag. (a) the perturbation (contoured) and the EPFD response (shaded), (b) the change in the streamfunction at 70 hPa.

(a) $\Delta$ NOGWD and Rayleigh drag (N)

(b) $\Delta$ residual streamfunction ($10^9$ kg s$^{-1}$), $\gamma=6$

- Direct
- EPFD+GWD
- EPFD
- NOGWD
- OGWD
Impact on index of refraction

Fig. 13. (a) and (b) the refractive index for wavenumber two (shading red) and the zonal-mean zonal wind (contours) for the integration with the Rayleigh drag and the NOGW, correspondingly. (c) the PV meridional gradient in the integration with the Rayleigh drag (shading red) and the change therefrom in the NOGW integration (contoured).
The wave forcings in ECHAM6

EPFD (m s\(^{-1}\) day\(^{-1}\))

OGWD (m s\(^{-1}\) day\(^{-1}\))

NOGWD (m s\(^{-1}\) day\(^{-1}\))